# EC 131 - Oligopoly - the Cournot Duopoly Model

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The Cournot duopoly model is characterized, for this course purposes, by the following assumptions:

- Two identical firms selling an homogeneous good (that is, consumers don't see any difference between the products from both firms)
- Constant marginal cost, which is the same for both firms
- Linear downward-sloping demand function

You will be given demand and cost structure as the following example:

$$TC = 2Q$$

$$MC = 2$$

$$P = 8 - Q$$

$$TR_1 = P \times Q_1$$

$$TR_2 = P \times Q_2$$

Note that:

- Calling $Q_1$ the quantity produced by firm 1 and $Q_2$ the quantity produced by firm 2, the total supply of the market is then $Q = Q_1 + Q_2$
- The demand function can, thus, be given as $P = 8 - Q_1 - Q_2$, which implies that total revenues for firms 1 and 2 can be written as $TR_1 = (8 - Q_1 - Q_2)Q_1$ and $TR_2 = (8 - Q_1 - Q_2)Q_2$

- Note that the revenue obtained by firm 1 depends not only on $Q_1$ but also on $Q_2$. This duopoly is, thus, a *game* played by the two firms.

Thus:

$$P = 8 - (Q_1 + Q_2)$$

and:

$$TR_1 = P \times Q_1 = (8 - (Q_1 + Q_2)) \times Q_1$$
$$TR_2 = P \times Q_2 = (8 - (Q_1 + Q_2)) \times Q_2$$

We can then derive the firms' Marginal Revenue functions:

$$MR_1 = \frac{\partial TR_1}{\partial Q_1} = 8 - 2Q_1 - Q_2$$
$$MR_2 = \frac{\partial TR_2}{\partial Q_2} = 8 - 2Q_2 - Q_1$$

Both firms have the objective of maximizing profits. Since the profit maximizing condition is $MC = MR$, we can apply that condition for each firm. Let's start with firm 1:

$$MC = MR_1 \implies 2 = 8 - 2Q_1 - Q_2$$

Remember that this is firm 1's profit maximizing condition. We should, thus, find firm 1's choice of $Q_1$ by isolating it from the equation above:

$$Q_1 = \frac{6 - Q_2}{2}$$

The equation above is called firm 1's *best-response function.* It gives, for each choice firm 2 may make of $Q_2$, the amount that firm 1 should produce to maximize its profits. By doing the same to firm 2, we get a similar result:

$$Q_2 = \frac{6 - Q_1}{2}$$

In order to find what firms will do, we must find the Nash Equilibrium. In the Nash Equilibrium, each firm is making the best choice given what others are doing. A situation in which this is true is when *all firms are making the choices given by their best-responses.* Thus, the two conditions below must be satisfied:

$$Q_1 = \frac{6 - Q_2}{2}$$
$$Q_2 = \frac{6 - Q_1}{2}$$

By replacing $Q_2$ in $Q_1$:

$$Q_1 = \frac{6 - \frac{6-Q_1}{2}}{2}$$

Rearranging:

$$Q_1 = \frac{6+Q_1}{4}$$
$$4Q_1 = 6 + Q_1$$
$$Q_1 = 2$$

And we can then find $Q_2$ by replacing $Q_1$ by 2:

$$Q_2 = \frac{6-2}{2} = 2$$

The Nash Equilibrium is, thus, for both firms to produce 2 units.

In order to find the prevailing price in that equilibrium, replace $Q_1$ and $Q_2$ in the demand function:

$$P = 8 - Q_1 - Q_2 = \$4$$

Profits are, thus, for each firm:

$$Profits_1 = P \times Q_1 - 2 \times Q_1 = 4 \times 2 - 2 \times 2 = \$4$$
$$Profits_2 = P \times Q_2 - 2 \times Q_2 = 4 \times 2 - 2 \times 2 = \$4$$

Total profits are, thus, \$8.

We saw in class that an oligopoly is a situation that can be represented by a Prisoner's Dilemma game. Being that the case, It should be the case that these firms could be better off if they could collude and form a cartel. Let's suppose that the two firms act as if they were a monopolist. The monopolist problem would be, thus:

$$TC = 2Q$$
$$MC = 2$$
$$P = 8 - Q$$
$$TR = P \times Q$$
$$MR = 8 - 2Q$$

Profit maximization ($MC = MR$) implies:

$$2 = 8 - 2Q \implies Q = 3$$

Price would then be:

$$P = 8 - 3 = \$5$$

Profits are, thus:

$$Profits = 5 \times 3 - 2 \times 3 = \$9$$

That is, a monopolist in that market would produce 3 units and obtain a profit of \$9. If the two firms produce $Q = 1.5$ they would, thus, obtain a profit of \$4.5 each, which is clearly higher than the duopoly profits.

Why don't they do that, then? It's because if each one is producing $Q = 1.5$, the firms have an incentive to cheat on the agreement. To see why, look at the best-response function for firm 1, for example. Suppose that they agreed on producing $Q = 1.5$ each. Then $Q_2 = 1.5$ and the best-response for firm 1 is:

$$Q_1 = \frac{6 - 1.5}{2} = 2.25$$

If fact, if $Q_1 = 2.25$ and $Q_2 = 1.5$, the price will be $P = 8 - 3.75 = \$4.25$. Profits for firm 1 would then become:

$$Profits_1 = 4.25 \times 2.25 - 2 \times 2.25 = \$5.0625$$

Which is, of course, higher than the profit obtained by following the cartel agreement.