Designing Heaven’s Will: Lessons in Market Design from the Chinese Imperial Civil Servants Match

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1. Introduction
Random Assignment Mechanisms

- Often, when deciding who should get what, we use randomization.
- The literature presents many properties that these procedures can have, and multiple mechanisms to implement them.
- Given some input, we get a distribution over outcomes, and draw an outcome from it.
USA military draft lottery
World cup groups lottery
Public housing assignments
Trust, transparency and simplicity

- When the stakes are high, drawing lots in public guarantees that the procedure and source of randomness is as promised.
- It gives legitimacy to allocation decisions when people distrust the institution or government, showing the absence of foul play.
- The mechanics of the procedure that is used for determining assignments are very simple.
Summary

• We provide a method which can be used in constrained assignment problems in a new and transparent way, drawing random matchings from urns in a public setting.

• Its design is informed by our historical analysis of the lots drawing method used for more than 300 years in imperial China to assign civil servants to jobs.

• The resulting procedure can be used in constrained assignment problems in a new and transparent way.

• We show how to use the lots drawing procedure in different types of problems, as:
  • Refugee matching,
  • Doctor specializations and hospitals needs,
  • Public housing.
Related literature

- **Random assignment mechanisms**: Random matching under dichotomous preferences, (Bogomolnaia, Moulin, 2014), Incompatibility constraints in kidney exchange (Roth et al., 2005; Akbarpour et al., 2016), Capacity constraints in refugee assignment (Andersson and Ehlers, 2017).
- **Market design in a historic context**: Debt clearing markets in pre-industrial Europe (Boerner, Hatfield, 2017), Papal conclaves (Mackenzie, 2017).
- **UEFA Champions League matching**: (Boczon and Wilson, 2018),
- **Maximum matching algorithms, and online matching algorithms**: Hall (1935), Berge (1957), Karp et al (1990)
- **Chinese examination system**: the abolishment and its impact on political stability (Bai and Jia, 2016)
The Chinese lots drawing procedure

- Introduced in 1594, and used until 1906 (fall of the empire).
- Every month, a set of civil servants would have to be matched to a set of jobs.
- **Rule of avoidance**: a worker cannot be matched to a job on his/her home province.
The Chinese lots drawing procedure

- Put all workers in one urn, and all jobs in another urn.
- Draw a candidate, then draw a job;
  - If the pair is compatible, then announce the match, and remove the pair from jars.
  - If the pair is not compatible, then put the job aside, and keep drawing until a compatible job is found. Then, announce the match and remove the pair from jars, and put back the incompatible job(s).
- Repeat the drawing until there are no unassigned jobs or candidates, or the unassigned jobs and candidates are incompatible.
Incompatibilities in the lots drawing procedure

Workers

$w_1$

$w_2$

$w_3$

Jobs

$j_1$

$j_2$

$j_3$
Incompatibilities in the lots drawing procedure

Workers

Jobs

$w_2$

$w_3$

$w_1$

$j_1$

$j_2$

$j_3$
Incompatibilities in the lots drawing procedure

Workers

Jobs

$w_2$

$w_1$

$w_3$

$j_1$

$j_2$

$j_3$
Remark
Unmatched candidates have to wait for two months.

Definition
A matching is efficient if there exists no other matching that matches more workers and jobs (efficiency = maximality).
A candidate who either draws an incompatible job or ends up with no compatible jobs left was suggested to exchange his incompatible job with another candidate who is matched with a compatible job in a mutually acceptable way.
A candidate who either draws an incompatible job or ends up with no compatible jobs left was suggested to exchange his incompatible job with another candidate who is matched with a compatible job in a mutually acceptable way.

There was however no indication that such an exchange was carried out. Hard to undo matches that are already made.
Inefficiency revisited - Prioritizing “hard-to-match” workers

Workers

\[ w_1 \]

\[ w_2 \]

\[ w_3 \]

Jobs

\[ j_1 \]

\[ j_2 \]

\[ j_3 \]
Inefficiency revisited - Prioritizing “hard-to-match” workers

Workers

$w_1$

$w_2$

$w_3$

Jobs

$j_1$

$j_2$

$j_3$
Inefficiency revisited - Prioritizing “hard-to-match” workers

Workers

Jobs

$w_2$

$w_1$

$w_3$

$j_1$

$j_2$

$j_3$
Inefficiency revisited - Prioritizing “hard-to-match” workers

Workers

Jobs

\( w_1 \)

\( w_2 \)

\( w_3 \)

\( j_1 \)

\( j_2 \)

\( j_3 \)
Proposition

For every realization of chance, prioritizing “hard to match” workers never leaves more workers unmatched than using the basic lots drawing procedure, and there are markets and realizations of chance where it leaves strictly less workers unmatched.
Historical evidence on prioritizing hard-to-match workers

Daoguang Year 4 [1824], it was approved, those who have home provinces to avoid draw first in the monthly appointment. If they still draw a job that needs to be avoided, remove this job and ask [the candidates] to draw another job. Until a [compatible] lot is drawn, then let those who do not need to avoid home provinces draw.

– Da qing hui dian (the Collected Institutes), vol 44, 1886 - 1899
• Contents of the urns were filled before the beginning of the procedure using a known pre-determined criterion, and remained unchanged except for the matches.

• Once a compatible match is drawn, the match is final and cannot be revised.
2. The generalized lots drawing procedure
Model setup

- A market is a triplet \( \langle W, J, C \rangle \) with:
  - A finite set of \( n \) workers \( W \);
  - A finite set of \( m \) jobs \( J \);
  - A compatibility correspondence \( C : W \rightarrow J \).
The generalized lots drawing procedure
The generalized lots drawing procedure
The generalized lots drawing procedure
The generalized lots drawing procedure
The generalized lots drawing procedure
The generalized lots drawing procedure

Workers

Jobs
The generalized lots drawing procedure

Workers

Jobs
Sequences of urns

\[ \varphi^W \]

\[ \varphi^W_1 \]

\[ \varphi^W_2 \]

\[ \varphi^W_3 \]

\[ \ldots \]

\[ \varphi^W_p \]

\[ \varphi^J \]

\[ \varphi^J_1 \]

\[ \varphi^J_2 \]

\[ \varphi^J_3 \]

\[ \ldots \]

\[ \varphi^J_q \]
The generalized lots drawing procedure

Definition

A sequence of urns is **efficient** if for every realization of chance, the outcome of the generalized lots drawing procedure using this sequence is efficient.
Minimum number of urns

There are markets that have no efficient sequence of urns using only one urn of workers.

There are markets that have no efficient sequence of urns using only one urn of jobs.
Theorem

For every market, there exists an efficient sequence of urns.
Definition

A sequence of urns satisfies **equal treatment of equals** if for any pair of workers $w, w' \in W$ where $C(w) = C(w')$, and any job $j \in J$, $w$ and $w'$ have the same probability of being matched to $j$, when using the generalized lots drawing procedure.
Theorem

For every market, there is a sequence of urns that is efficient and satisfies equal treatment of equals.
3. Families of problems
Multi-hierarchical constraint
Multi-hierarchical constraint

- **Example**: matching refugees to hosting families
  - Matching each worker to a task, with time limitations,
  - A refugee family needs:
    - $x$ beds,
    - $y$ primary school spots,
    - $z$ secondary school spots...
Multi-hierarchical constraint
Multi-hierarchical constraint

\[ \varphi_W^W \]

\[ \varphi_1^W \]

\[ \varphi_2^W \]

\[ \varphi_3^W \]

\[ \varphi_J^J \]

\[ J_1, J_2, J_3 \]

\[ J_4, J_5, J_6, J_7 \]

\[ J_8, J_9 \]

\[ J_1, J_2, J_3, J_4, J_5, J_6, J_7, J_8, J_9 \]

\[ J_1, J_2, J_3, J_4, J_5, J_6, J_7, J_8, J_9, J_{10} \]
Joint 2-constraints

\[ \tau_1, \tau_2 \quad \tau_1, \tau_3 \quad \tau_1, \tau_4 \quad \tau_2, \tau_3 \quad \tau_2, \tau_4 \quad \tau_3, \tau_4 \]
Example

*Let doctors have specializations in either Orthopedics ($\tau_O$), or Neurology ($\tau_N$). In addition to that, some doctors may also have a certificate in Family Medicine ($\tau_F$) or Surgery ($\tau_S$). Hospitals have jobs for Orthopedics ($J^O$), Neurology ($J^N$), and also positions that require only a certificate in Family Medicine ($J^F$) or Surgery ($J^S$).*
Joint 2-constraints

\[
\begin{align*}
\varphi_1^W & = \tau_O \quad \tau_N \quad \tau_F \quad \tau_S \\
\varphi_2^W & = \tau_{O \cup N} \quad \tau_{O \cup F} \quad \tau_{O \cup S} \\
\varphi_3^W & = \tau_{N \cup F} \quad \tau_{N \cup S} \\
\varphi_4^W & = \tau_{F \cup S} \\
\varphi_1^J & = \varphi_1^J \\
\varphi_2^J & = \varphi_2^J \\
\varphi_3^J & = \varphi_3^J \\
\varphi_4^J & = \varphi_4^J
\end{align*}
\]
Joint 2-constraints: Public Housing
Joint 2-constraints: Public Housing

\[ \varphi^W_1 \quad \varphi^W_2 \quad \varphi^W_3 \]

\[ I_1 \quad I_3 \quad I_5 \quad I_6 \]

\[ \varphi^J_1 \quad \varphi^J_2 \quad \varphi^J_3 \quad \varphi^J_4 \]

\[ H_1 \quad H_2 \quad H_3 \quad H_4 \]
4. Conclusion
• In many real-life allocation problems, transparency and simplicity is very important.

• Drawing lots: simple, transparent and historically robust procedure.

• Generalized lots drawing procedure
  - Always yields maximum matchings,
  - There are always solutions satisfying equal treatment of equals.

• We provide the sequence of urns to use for some general families of problems.
Thanks!
Inefficiency in the lots drawing procedure - Example

Workers

\[ n \]

\[ n \]

\[ 2n \]

Jobs

\[ n \]

\[ n \]

\[ 2n \]
Inefficiency in the lots drawing procedure - Example

Workers

Jobs

$n$

$n$

$n$

$2n$

$2n$
Inefficiency in the lots drawing procedure - Example

Workers

Jobs

$2n$

$n$

$n$

$n$

$2n$
Inefficiency in the lots drawing procedure - Example

Workers

Jobs

$n$

$2n$

$n$

$2n$

$n$